Practice Points NUMBER 18

Classical Structural Analysis Methods for Historic Structures

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The architecture, engineering, and preservation professions have carried out a vigorous and long-standing discussion of the design of the structural aspects of historic buildings, of their construction, and of methods for repairing, rehabilitating, and reconstructing them. Publications on applying modern structural analysis in the interpretation of historic buildings and structures are also widely available. However, such analyses only rarely mention the type of analysis conducted at the time of original construction of the property under investigation. Significantly less published information is available on the synthesis of design methods used in the production of the original buildings, bridges, and other structures.[1] Although there is recognizable value in applying modern, better-understood analysis methods to the structural analysis of historic structures, there is similar value in reviewing the original design intent and the methods used to achieve these objectives. For example, an understanding of the applications of graphical analysis to the determination of proper forms for trussed, vaulted, and domed structures has enriched the history of the design of these structures and has improved the ability of the engineering profession to manage such structures effectively.

The analysis and design methods for structural systems—arches, beams, frames, columns—as used in early American architecture and engineering can be broadly classified into three groups: empirical, analytical, and graphical.[2] The use of empirical methods is universal to all engineering and can be recognized in all ancient works of engineering and architecture. The word "empirical" derives from an ancient Greek word meaning "experience," and empirical design can be understood as experience-based design. It involves the use of simple proportional ratios, geometrical rules, the scaling up of previous successful works, cautious experimentation and careful deviation from previously established rules, and the observation of standard or minimum sizes and thicknesses. Examples of empirical design of wood and masonry structures are presented below.

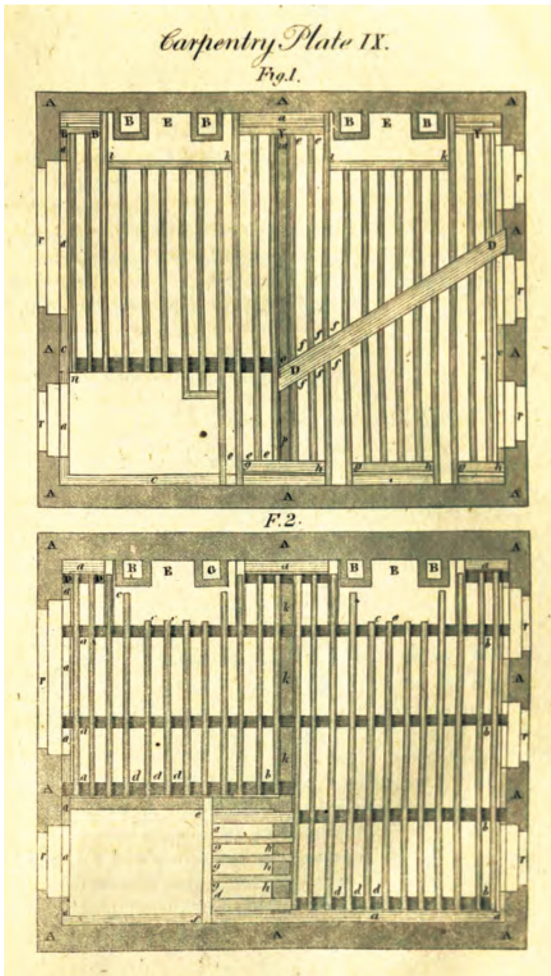

Fig. 1. *Wood floor system showing the configuration of girders, binding joists, and bridging joists in a wood floor from Peter Nicholson,* Nicholson's New Carpenter's Guide *(London, 1828). Courtesy of Pennsylvania State University Libraries Special Collections.*

Analytical design was first successfully applied to structural design in the early to mid-nineteenth century and gained increasing importance until, at the present time, it is practically the only trusted method of engineering design. Analytical design involves the determination of forces and stress in a material through mathematical formulas. This method is related to the experimental determination of the force in a member and to experimental verification of the theory used to calculate the stresses. A variety of analytical methods that were applied effectively to the design of structures appeared in the late nineteenth and early twentieth centuries. Many of these methods evolved into the contemporary methods of structural analysis and design, while many others became obsolete.

Graphical methods are an alternative to analytical design, in which the equations of equilibrium and the resistance of materials are solved by the construction of scaled diagrams.

Empirical Engineering before 1865

The empirical procedures employed during the first half of the nineteenth century are discussed and illustrated in manuals of carpentry, stonework, and, occasionally, ironwork. Peter Nicholson's carpentry book, *Nicholson's New Carpenter's Guide*, can be used as an example, although a great number of alternative sources are available.[3] According to Nicholson, the sizing of floor joists, floor girders, and roof trusses is an enterprise that is accomplished descriptively and by recourse to examples (Fig. 1).

Wood. Such empirical rules were widely used in carpentry for sizing floor joists, girders, posts, and other components of wood framing. Other examples of the empirical engineering of floor systems are available in other nineteenth-century carpentry manuals. Such systems were often complex, with floorboards running diagonally, bridging joists transverse to them, binding joists supporting the bridging joists, and, often, girders supporting the bridging joists. Nicholson, an English writer whose methods were widely used in the U.S., estimates the size of all these elements by tables of span and size. A span-to-depth ratio for binding joists can be inferred from these tables: 15:1–16:1 for bridging joists of fir and 16:1–17:1 for oak. For bridging joists, which are continuous over binding joists, the span-to-depth ratio approaches 20:1 for the longer spans. The depths for girders, similarly, are approximately 14 times the span for shorter spans, such as 12 feet, and as much as 18 feet for longer spans. The widths of the girders increase in proportion to the depth, with a normative depth-to-width ratio of 1.2:1 for girders and 2:1–3:1 for joists.[4]

In other tables, Nicholson designs wood posts to follow a similar rule without regard for the anticipated load on the post but sized solely by length. A post 8 feet tall requires a 6-by-10-inch timber, while a 20-foot post requires a 12-by-16-inch timber. This is equivalent to a height-to-width ratio of 16:1–20:1. For all of the proposed sizes, the height-to-width or span-to-depth ratio increases slightly as the member increases in length.[5]

Metals. The engineering of metals in this time period involved mostly blacksmithing and iron castings. The predominant engineering materials were wood and masonry, but wood needed connections in the form of straps, bolts, castings, rods, and other devices. All of these provisions were sparing in their use of metal, which could be produced only in limited supplies. Cast-iron columns were a more frequently occurring member; cast-iron girders were far less commonly used. The engineering of these elements included empirical rules for sizing, as well as some early analytical formulas for the strength of metal columns, such as Hodgkinson's Rule for cast-iron columns. This rule is empirical in the sense of being based on tests without considering an underlying theory and contains empirical modifications for a shorter column.[6]

Bridges. A number of empirically engineered, wood truss bridges were constructed up through the 1850s in the northeastern U.S. Among these is, for instance, Theodore Palmer's truss bridge over the Merrimack River in Newburyport, Massachusetts.[7] Palmer had no training as an engineer, but as an empiricist, he apparently based his truss design on an established precedent, in this case a drawing in Palladio's *The Four Books on Architecture*.[8] He respected the proportions of these trusses and added additional strengthening below the trusses.[9]

A prominent engineer who left his mark on structures both before and after 1865 was Herman Haupt. His book, *General Theory of Bridge Construction*, employs calculation methods for determining the size of bridge members, although the configurations are determined by experience; significant graphical and empirical design elements also appear in his book.[10] During the Civil War, Haupt's fame was enhanced by the construction of hastily improvised bridge repairs that appear to be based solely on empirical considerations. After the war, he reverted to analytical design methods, such as the iron truss bridge crossing the Susquehanna River at Rockville, Pennsylvania.[11]

Haupt's method of analyzing arches may be characterized as "semi-empirical," as he supposes a collapse mechanism and completes a simple analysis of moment equilibrium, in terms of available resistance, on the basis of this collapse mechanism.[12] His work on wooden bridges consists of laborious calculations of vertical and horizontal strains, interspersed with discussions on practical matters involving deviations from these rules (Fig. 2).[13]

Engineering after 1865

As a result of rapid and large-scale advances in engineering during the Civil War and then during the late nineteenth century, a great variety of engineering texts was published. Examples include a book by George Fillmore Swain, professor of engineering at the Massachusetts Institute of Technology, and another by Augustus Jay Du Bois, professor of civil engineering at Yale University's Sheffield Scientific School. The prefaces to these books describe the methods employed as scientific, and the authors invoke science as the basis of the methods of analysis and design that they are describing.[14] The authors alternate between empirical discussions of standard forms of construction for bridges, buildings, or foundations and presentation of theories and methods of engineering analysis. Swain's lecture notes, published in mimeograph form and available today through a number of sources, are particularly rich in practical knowledge of bridge construction.[15]

Wood. Much of the analytical material for wood structural design was indirect, such as the determination of design values. A widely used design value for resistance to bending of wood beams, for example, was based on the breaking of a billet that was 1 inch square and 12 inches long, which is denoted *B* in later editions of Robert Hatfield's 1874 book *Theory of Transverse Strains*. Given this value, empirical rules were developed for assessing the strength of a wood beam. These rules still use mixed units, feet, inches, and a constant whose units are unrelated, which is one of the characteristics of empirical design. An example of such a rule is

$$180cL^2 = Bbd^2$$

where *c* is spacing; *L* is the span (in feet); *B* is the billet; *b* is the width of joist; and *d* is the depth (in inches).[16]

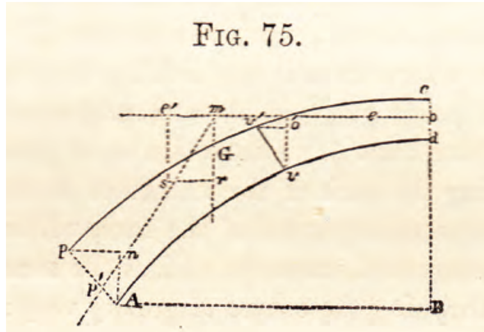

Fig. 2.
Equilibrium analysis of an arch from Herman Haupt, A General Theory of Bridge Construction *(New York, 1853). Haupt assumes a fracture joint, directions and locations for the crown thrust, and the abutment reaction and determines the susceptibility to failure at the joint by equilibrium. Courtesy of Pennsylvania State University Libraries.*

Masonry. Masonry arches are described in contemporary treatises, some of which are listed below, using analytical procedures for the design of the structure. The analysis methods were similar in that they all employed equilibrium calculations at various cross-sections of the arch. The equilibrium calculations were used to find the axial force and eccentricity of the internal forces in the arch. Having the axial force and eccentricity in hand, the designer could find the stresses due to axial force and bending moment and then use these stresses in the arch ring to evaluate the capacity of the arch. Allowable stresses were published in various sources, notably in Ira Osborn Baker's *A Treatise on Masonry Construction*, first published in 1889, and incorporated a factor of safety of five or greater.

Extensive methods are also presented in Baker and in Swain for the determination of the correct line of internal pressure, using least pressure, least error, etc.[17] Because such methods required laborious calculations, there was a general preference for empirical methods for the design of bridge arches and only the occasional use of graphical methods. For masonry arches in buildings, empirical methods were primarily used.[18]

Table 1. Allowable Stresses in Masonry Arches[19]

Type of Masonry	Safe Pressure Tons per square foot	 Pounds per square inch
Rubble	10 to 15	140 to 200
Squared-stone	15 to 20	200 to 280
Limestone ashlar	20 to 25	280 to 350
Granite ashlar	25 to 30	350 to 400
Concrete	30 to 40	400 to 550

Iron and steel. Analytical methods are also used for the description of metallic structures in steel or in iron. The analysis of trusses depends on understanding the flow of forces through the truss. Bridge trusses are usually calculated on the basis of the overall shear in the truss for various load states. This shear can be carried through the truss, as in the indexing method for the analysis of trusses, and it can be resolved into the forces in each of the bars of the truss. An example of the application of the indexing method for a 10-panel truss is shown in Figure 3, where a dead load of 32 kips bottom chord and 10 kips top chord and a live load of 60 kips per panel are determined on the basis of self-weight and locomotive loading per lineal foot times the length of the panel. For full loading, the total shear at each panel is found and written on the diagonal bar of the diagram. The panel indices are then summed to find the chord forces. To find the actual bar forces from the index forces, the index force is multiplied by the panel width-to-height ratio for the chords, and by the diagonal length-to-panel-height ratio for the diagonal braces (Fig. 3).[20]

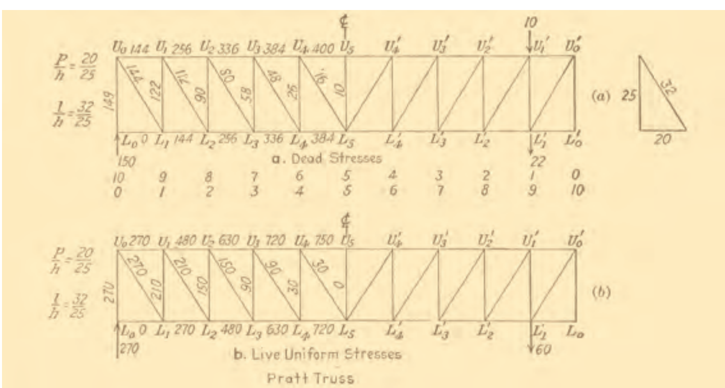

Fig. 3.
Indexing analysis of a bridge truss from G. F. Swain, Structural Engineering, *vol. 3 (New York, 1923).*

Table 2. Coefficients *a* for the Application of the Rankine-Gordon Formula

Column Type	*f*	*a*
Wood (round)	1,200	250
Wood (rectangular)	1,200	250
Iron (W-shape)	30,000	1,200
Iron (round)	27,000	1,200
Iron (box)	27,000	2,000
Iron (cruciform)	22,000	1,000

Figure 4 shows a four-span continuous girder bridge, dated 1925, similar to the types of construction described in the works of the late nineteenth century. Continuous girders and trusses were analyzed by “Clapeyron’s theorem” or the “theorem of three moments.” Based on continuity of a beam at an internal joint, a formula involving three moments—one at the bending moment at the joint in question, one at the next support to the left, and one at the next support to the right—can be written. These equations allow for the solution of the support moments, as the number of equations that can be written is equal to the number of unknown support moments (Fig. 4).

The internal-support moments (M_1, M_2, and M_3) can be found by writing equilibrium/compatibility equations at successive joints, as in the following equations for a four-span girder with four equal spans (*L*), with each span subjected to a uniform dead load (*w*). The equations become more complicated for cases of variable or concentrated loading or for unequal spans.

$$4M_1L + M_2L = \frac{wL^2}{2}$$

$$M_1L + 4M_2L + M_3L = \frac{wL^2}{2}$$

$$M_2L + 4M_3L = \frac{wL^2}{2}$$

As the number of spans becomes larger, the difficulties in solving for the unknown moments become greater.[21]

Wood, iron, and steel columns. Column design was the result of the application of a semi-empirical formula known as the “Rankine-Gordon formula” or “Gordon’s formula.” It has two forms, one rational, based on the residual strength of a column after deducting the stresses produced by bending, which is due to Rankine, and a formula similar in appearance, which uses arbitrary values of empirical constants in order to tune the formula.[22] The Rankine-Gordon formula for allowable axial stress in a column is

$$F_a = \frac{f}{1 + \frac{l^2}{ad^2}}$$

where F_a equals the allowable compressive stress in column; *l* is the length of column between weak axis supports; *a* is the constant determined from Table 2 at left; and *d* is the characteristic least diameter of column.

The numerator *f* represents the maximum compressive stress in the material, usually with a factor of safety of four applied, while the coefficient *a* in the denominator is empirical, based on differing materials and cross sections, according to Table 2. In the table, iron refers to wrought iron, except for the round column, which may be of wrought iron or cast iron.

So, for example, a wide, flange-shaped, wrought-iron column measuring 8 inches deep with an 8-inch flange width, with a height of 120 inches and a maximum compressive stress (including safety factor) of 15,000 pounds per square inch, has an allowable axial stress of 12,600 pounds per square inch.

Graphical Analysis

Later nineteenth-century engineering is also distinguished by the widespread application of graphical methods of analysis and design (Fig. 5).

Graphical analysis of trusses was extensively applied to bridge trusses and to roof trusses for buildings. William Merrill in *Iron Truss Bridges for Railroads* describes a graphical procedure for finding the forces in every part of a truss based on the loading at a single joint.[23] An initial graphical solution of the resultant forces to the right and to the left of the loaded point is completed by the simple expedient of drawing a scaled line to the two supports of the bridge. When the direction of this resultant is determined on each side, the resultant can be decomposed graphically into the force components in each of the bars of the truss on the left and on the right side. The application of this method to bridge trusses is shown in Figure 5. A complete truss solution by this method would necessarily require superposition of the results for all of the loaded joints in the truss.

Graphical analysis of trusses in buildings was very widely used to find the forces in the bars of a truss. The method, which involved the development of a scaled drawing of the forces in the bars of the truss, is illustrated in Figure 6 reproduced from William Wolfe's *Graphical Analysis* of 1927. In Wolfe's book, Figure 258 represents the truss; Figures 259 through 262 represent the equilibrium of the forces at each joint; and Figure 263 represents the complete force diagram of the truss.[24] Figure 255 shows the truss loaded at three panel points by concentrated forces on the top chord. Figures 256 through 262 show the successive construction of a vector diagram of the forces at each joint, with the joints solved in the order of the Roman numerals I through VII. The final figure in the set, Figure 263, shows the combined solution of the truss in a single diagram. With such a diagram, it is possible for a present-day engineer to determine the forces used in the original design of trusses far more complicated than the one depicted here.

Graphical analysis was also used in various forms to determine the magnitude and the eccentricity of the internal force in a given arch by constructing a line of internal pressure and verifying its location within the arch. This procedure is illustrated in Figure 7, which is taken from Frank Kidder's *The Architects' and Engineers' Pocket-Book* of 1886.[25] In the graphical procedure for the arch, a scaled diagram of the loads is assembled into a vertical line (AE), and a pole (O) is chosen. Lines 1 through 10 radiating from the pole are used to decompose each of the loads into a pair of forces. In plotting these forces on the arch diagram, the components of each load (block weight) must intersect on the line of action of that load (the vertical line through block centroid); thus a chain of forces can be drawn on the arch diagram (denoted as n through a in Figure 7) to represent an equilibrium configuration consistent with the set of loads. In order to make this curve fit the shape of the arch,

Fig. 4.
Sellwood Bridge, Willamette River, Oregon, 1925. HAER OR-103. This bridge, built in 1925 with four continuous girder spans, was probably analyzed using Clapeyron's theorem. Similar multiple-span truss bridges were occasionally used in the nineteenth century and analyzed in this manner.

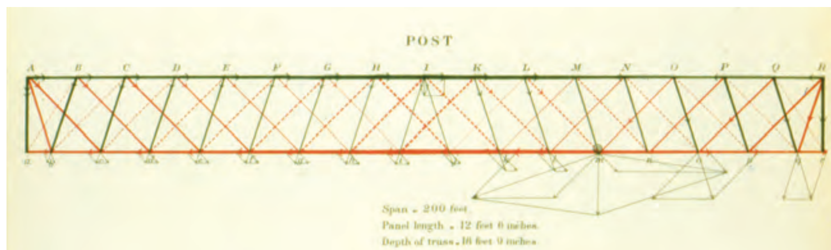

Fig. 5.
Analysis of a truss bridge from William Merrill, Iron Truss Bridges for Railroads*, 4th ed. (New York, 1878). For a unit load at lower chord joint m, lines mA and mR are drawn to determine the direction of the reactions. The reactions are decomposed into forces parallel to the bottom chord and to the tie on each side of the joint. This process of decomposition of forces is repeated for the remaining joints on the bridge. Courtesy of Pennsylvania State University Libraries Special Collections.*

the resultant (vertical through C) of all the loads is found and used to construct the horizontal internal force at the crown (crown thrust AC) and the direction and location of the force at the support (BC). The internal forces in the arch can be rectified by constructing a new pole at P or by a more complex geometrical procedure. These forces can be checked against the middle third or other rule for the safety of the arch. It is

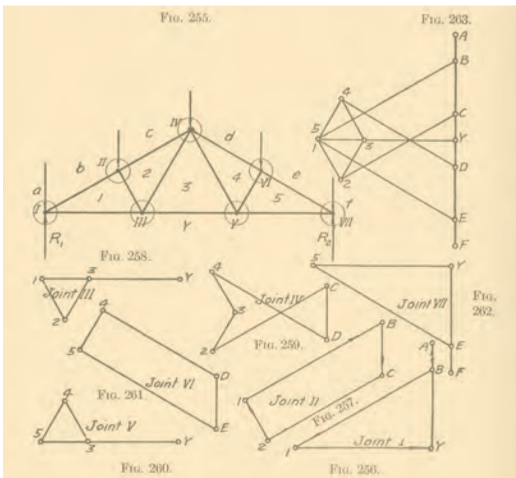

Fig. 6.
Graphical analysis of a roof truss from William Wolfe, Graphical Analysis *(New York, 1921).*

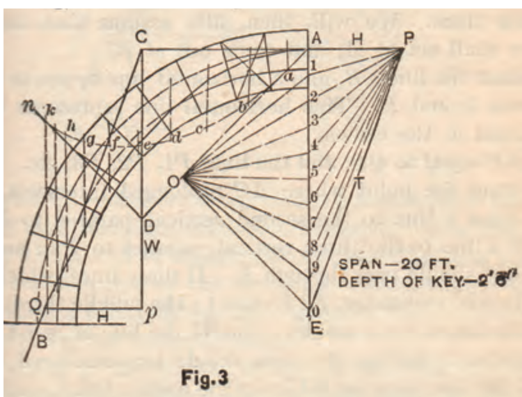

Fig. 7.
Graphical analysis of an arch from Frank Kidder, The Architects' and Engineers' Pocket-Book, *3rd ed. (New York, 1886).*

more commonly accepted today that the line of pressure can diverge much closer to the interior or exterior face of the arch.[26] The combination of this understanding and the graphical construction of the line of pressure of an arch can equip preservation engineers to make reasonable assessments of the capacity of an arch (Fig. 7).

Modern Uses of Classical Methods

There are potential modern uses of many of the methods outlined in this brief discussion, especially for preservation engineers who may want to reproduce the analysis used by the designers of a bridge or building. At the very least, the use of empirical formulas for the determination of appropriate sizes is useful in estimating sizes or in making an initial determination that a member is sized correctly. These methods are especially valuable in assessing wood-framed and masonry structures. The indexing methods of truss analysis may be very useful to a preservation engineer in estimating the capacity of a truss. Graphical analysis of trusses is similarly revealing of truss behavior and is of potential value to a preservation engineer. The analysis of arches is simpler and often more effective by the application of graphical methods. Moreover, graphical analysis is more revealing of arch behavior than any up-to-date analytical method.

Conclusion

The time period of the industrialization of North America coincides with a shift from strictly empirical methods of engineering analysis to combinations of empirical and analytical methods. By the late nineteenth century, engineers practiced self-consciously "scientific" methods of engineering, which had a significant component of practical empirical knowledge but which also involved calculations based on theories of mechanics. The application of analytical methods pertained foremost to the analysis and design of trusses and beams, while columns were subjected to a semi-empirical method, the Rankine-Gordon formula. The design of masonry load-bearing walls and arches in buildings remained staunchly empirical through the early twentieth century and the eventual disuse of this form of construction.

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Notes

1. Some of this article refers to the author's book *Engineering Iron and Stone* (Washington, D.C.: ASCE Press, 2015), which contains information about all aspects of late nineteenth-century and early twentieth-century design. Important preservation publications up to 1980 are listed in R. E. Haynes, *Historic Preservation Bibliography* (Washington, D.C.: National Park Service, 1979). Since the *Preservation Briefs* were written in the 1970s, the volume of literature on technical aspects of historic preservation has increased dramatically, with several periodicals, including the present *APT Bulletin*, devoted to the exploration of technical issues, including structural, in historic preservation.

2. Boothby, *Engineering Iron and Stone.*

3. Peter Nicholson, *An Improved and Enlarged Edition of Nicholson's New Carpenter's Guide: Being a Complete Book of Lines, for Carpenters, Joiners, and Workmen in General, on Methods Entirely New, Founded on Geometrical Principles* (London: Jones and Company, 1828). Benjamin Hale, *Introduction to the Mechanical Principles of Carpentry* (London: Richardson and Lord, 1827). Thomas Tredgold, *Elementary Principles of Carpentry* (New York: E. & F. N. Spon, 1888).

4. Nicholson, 55–56.

5. Nicholson, 53.

6. Hodgkinson's Rule, described in Boothby, can be repeated in part here. The breaking load (in pounds) for a solid, round cast-iron column is taken as 33,380 ($d^{3.76}/L^{1.7}$), where *d* is the diameter in inches and *L* is the length in feet. Hollow-column capacity is the difference between the solid column and an equivalent column of the diameter removed. Column capacity is increased for shorter columns.

7. Frank Griggs, "Newburyport Bridge," *Structure Magazine*, June 2013, 26–28.

8. Palladio's truss designs are illustrated in, for instance, Andrea Palladio, *The Four Books on Architecture,* trans. Robert Tavernor and Richard Schofield (Cambridge, Mass.: MIT Press, 1997), 176–179.

9. An image of Palmer's bridge at Newburyport, Massachusetts, is available courtesy of *Structure Magazine* at https://www.structuremag.org/wp-content/uploads/0613-hs-1.jpg.

10. Herman Haupt, *General Theory of Bridge Construction* (New York: D. Appleton and Company, 1853).

11. An image of the iron bridge at Rockville is available courtesy of the Historic Bridges website at https://historicbridges.org/pennsylvania/rockville/iron_large.jpg. It was replaced in 1900 by the present stone-arch bridge.

12. Haupt, 125–140.

13. See Haupt, page 8, for his critique of the analytical procedure of equilibrating arch forces. His semi-empirical procedure is outlined on pages 125–140.

14. G. F. Swain, *Stresses, Graphical Statics, and Masonry* (New York: McGraw Hill, 1927), 27. A. J. Du Bois, *The Strains in Framed Structures*, 5th ed. (New York: John Wiley and Son, 1890), 3.

15. G. F. Swain, *Notes on the Theory of Structures,* 2nd ed. (Cambridge, Mass.: Massachusetts Institute of Technology, 1896). This book consists of Swain's mimeographed course notes from lectures presented within the Massachusetts Institute of Technology's Department of Civil Engineering, in Cambridge, Massachusetts, in 1896.

16. R. G. Hatfield, *Theory of Transverse Strains* (New York: Wiley and Sons, 1877), 89. Hatfield and other authors also develop constants for the evaluation of members in bending based on the strength of a 1-inch-square and 18-inch-long billet and apply this constant through rules to the determination of beam and joist sizes.

17. I. O. Baker, *A Treatise on Masonry Construction*, 9th ed. (New York: McGraw Hill, 1907), 444–494. Swain, *Stresses, Graphical Statics, and Masonry*, 411–425. Swain, *Notes on the Theory of Structures,* 123–133. Table 1 is based on Baker, 151.

18. Rankine's rule, the most widely used empirical method, required only the intrados radius of the arch for the determination of its thickness; that is, the arch thickness in feet is equal to 0.35 times the square root of the intrados radius in feet. Alternative rules were available, summarized in the cited works of Swain and Boothby.

19. Table 1 refers to allowable stresses in masonry arches after Baker, 151.

20. Swain, *Structural Engineering*, vol. 3 (New York: 1923), 89. Boothby, 85–86.

21. Mansfield Merriman, *On the Theory and Calculation of Continuous Bridges* (New York: D. van Nostrand, 1876).

22. J. D. Crehore, *Mechanics of the Girder* (New York: John Wiley and Sons, 1886), 299.

23. William Merrill, *Iron Truss Bridges for Railroads,* 4th ed. (New York, 1878).

24. For a more complete review of this procedure, see Boothby, chapter 12.

25. Frank Kidder, *The Architects' and Engineers' Pocket-Book,* 3rd ed. (New York: John Wiley and Sons, 1886). For a more complete review of this procedure, see Boothby.

26. Jacques Heyman, *The Stone Skeleton* (Cambridge, Mass.: Cambridge Univ. Press, 1995).

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